Name:___________________________

SM3 7.1 Exponential Graphs

The core of an exponential function is having a variable as the exponent of a base.

$f(x) = 2^x$ — x is the variable as an exponent; 2 is the base

We can produce a table of values to examine the behavior of the exponential function, just like we did with polynomial functions.

x	$f(x)$
-4	$\frac{1}{16}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16

Raising our base to a negative power results in the power being in the denominator.

Any number raised to 0 power is 1.

Notice that the $f(x)$ values, $2, 4, 8,$ and 16 are all powers of the base, 2.

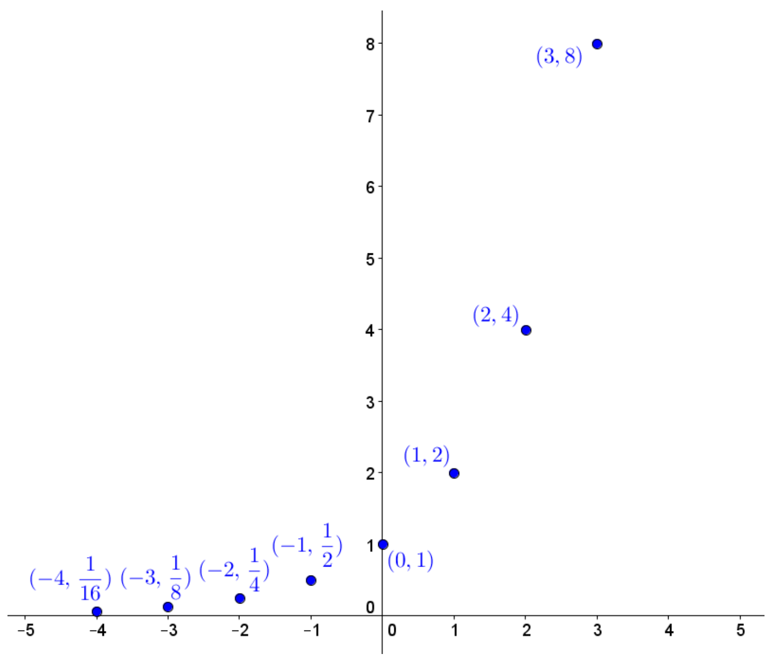
Plotting the points of the exponential function, we can see a pattern for the end behavior.

To the left, the values are getting small, each being half as tall as its neighbor on its right.

To the right, the values are getting large, each being twice as tall as the previous. The values are so large that $(4,16)$ would require a much larger field to include.

$f(x)$ has no x-intercept.
$f(x)$ has y-intercept of $(0,1)$.

All real numbers are included in the domain. The range contains all of the positive numbers. $f(x)$ is strictly increasing.



Go ahead, connect the dots to draw the curve $f(x) = 2^x$.

Did we really need to make a table of 9 values to see the curve? Fortunately, no! As long as we can determine the end behaviors and central value of the exponential function, we're safe to assume that the curve is predictable and smooth.

Let's graph the same function again, but this time doing the minimum amount of work to completely understand the behavior of the function!

<u>Example</u>: Graph $f(x) = 2^x$

The graph is not shifted left or right, so the x-value of the central value is $x = 0$. Since $f(0) = 2^0 = 1$, the central y-value is 1. So, $(0,1)$ is on the graph.

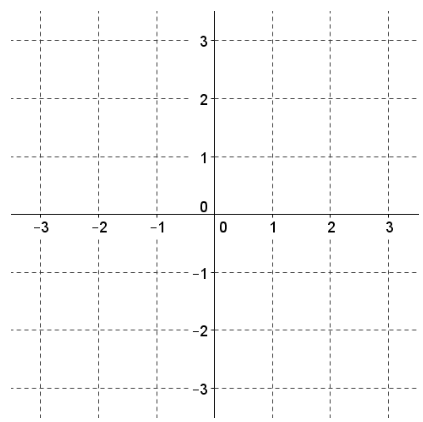
We just need to find the path left and right of $x = 0$ to understand the behavior of the function, so let's test the values $x = -1$ and $x = 1$.

$f(-1) = 2^{-1} = \frac{1}{2}$. So, $(-1, \frac{1}{2})$ is on the graph.
$f(1) = 2^1 = 2$. So, $(1,2)$ is on the graph.

All of the work on the left should be done on scratch paper or in the student's mind. Plotting points by evaluating a function at a few x-values should be a brief mental exercise at this point.

Step 1) Plot the points: $\left(-1, \frac{1}{2}\right)$, $(0,1)$, and $(1,2)$.

Step 2) Draw a continuous curve that intersects the points that continues to change exponentially using the three plotted points to determine the pattern*.



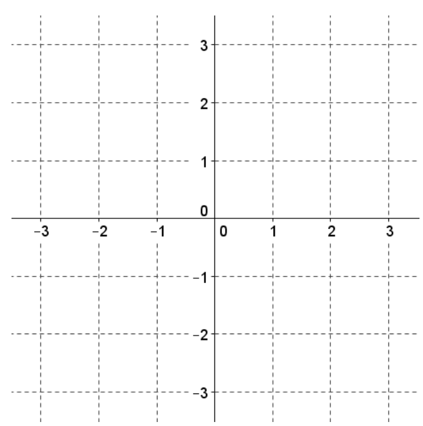
*In this case, moving left causes the y-value to be halved, moving right causes the y-value to be doubled.

<u>Example</u>: Graph $f(x) = 3^x$

Let's move a little faster on this one.

Step 1) Determine the three points that need to be plotted.

Step 2) Draw the exponential curve.



Basic Transformations

As you recall from secondary math 2, a function can be altered by:

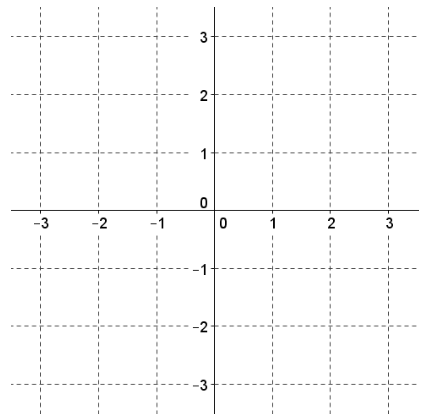
- changing the signs of the components of the function causing the function to flip
- adding a constant to the function causing the function to shift vertically
- adding a constant to the variable within the function causing the function to shift horizontally
- multiplying the function by a coefficient causing the function to scale.

Let's experiment with transforming an exponential curve:

Example: Graph $f(x) = -2^x$

Step 1) The variable doesn't have anything added to it, so we'll still use $x = 0$ as our central x-value. When evaluating the function at the appropriate x-values, be certain to multiply by the – sign; all of the y-values will be negative.

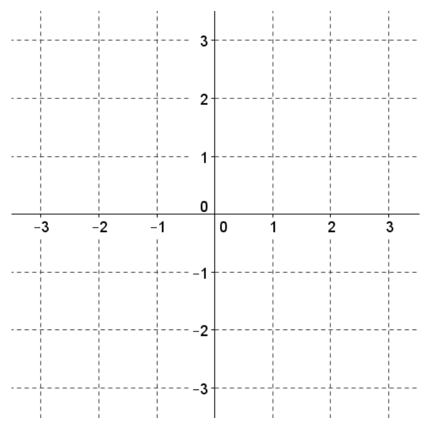
Step 2) Draw the exponential curve that fits the points.



Example: Graph $f(x) = 2^x - 1$

Step 1) The variable doesn't have anything added to it, so we'll still use $x = 0$ as our central x-value. When evaluating the function at the appropriate x-values, be certain to subtract 1 to select the right y-values.

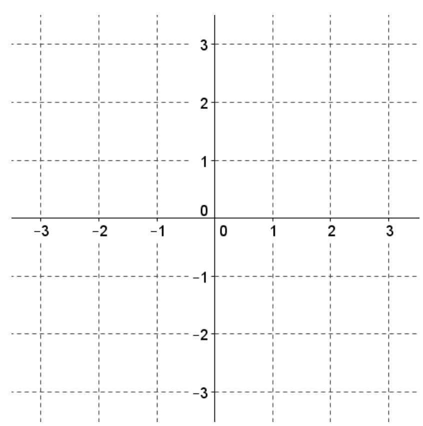
Step 2) Draw the exponential curve that fits the points.



Example: Graph $f(x) = 2^{x+1}$

Step 1) The variable has a $+1$ added to it, which shifts the function left by 1, so we'll use $x = -1$ as our central x-value. Evaluating the function at the x-values left by 1 and right by 1 from $x = -1$.

Step 2) Draw the exponential curve that fits the points.



HW4.1

Graph the following. Label the central point as well as two points on either side of it.

1. $y = 2^x$

2. $y = -4^x$

3. $y = 5^{-x} + 1$

4. $y = -2(4)^{x-1}$

5. $y = -(2)^{x+2}$

6. $y = 2^x + 1$

7. $y = -\left(\frac{1}{4}\right)^x$

8. $y = 3\left(\frac{3}{2}\right)^x$

9. e^{-x}

10. $y = -e^x$

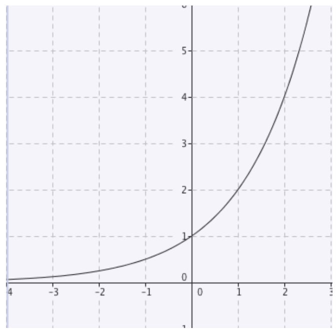
11. $y = 3 + e^x$

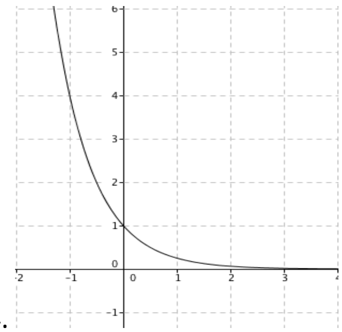
12. $4e^{x-1}$

For problems 9-18 find the listed properties. You may graph the function if you find that it helps you to see the properties:

a. domain *b. range* *c. intervals of increase and decrease*

d. y-intercept *e. end behavior*

13. 

14. 

15. $y = 3^x$

16. $y = 3^{-x}$

17. $y = -3^x$

18. $y = -3^{-x}$

19. $y = -2(3^x)$

20. $y = 2(3^x)$

21. $y = a(b^x)$, a and b are natural numbers greater than 1

22. $y = -a(b^x)$, a and b are natural numbers greater than

Find the average rate of change on the given interval.

23. $y = 5(2^x)$, $[3,5]$

24. $y = e^{2x}$,$(0,3)$

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